POLYNOMIALS

Polynomial is a mathematical expression of one or more algebraic terms each of which consists of a constant multiplied by one or more variable raised to a non-negative integral power such as (ax^2+bx+c)

Kinds of Polynomial :⟶

* A monomial is a polynomial with only one term, such as $3x$, $4xy$, 7 and $3x^2y^{34}$ etc
* A binomial is a polynomial with exactly two terms such as $x+3$, $4x^2+5x$, $x+2y^7$ etc
* A trinomial is a polynomial with exactly three terms such as $+x^4+3x^3-2$, $3x^2+3x+2$ etc

Another special kind of polynomial is quadratic equation which is polynomial of degree 2. It looks like ax^2+bx+c where a, b and c are constant.

Another one is cubic polynomial. which has degree 3. It looks like ax^3+bx^2+cx+c

Some more examples

$3x^2+2$ ⟶ Quadratic, $3x^2+2$ ⟶ Cubic polynomial.

Zero polynomial :→ The constant polynomial 0 is called the zero polynomial. the degree of zero polynomial is not defined.

Exercise 2.1

Q 1) Which of the following expression are polynomials in one variable and which are not? state reason for your answer.

i) $4x^2 - 3x + 7$

Sol:→ Polynomial in one variable 'x'

ii) $y^2 + \sqrt{2}$

Sol Polynomial in one variable 'y'

iii) $3\sqrt{t} + \sqrt{2}$

Sol It is not a polynomial as power of t is not a whole number.

iv) $y + \frac{2}{y}$

Sol:→ This is not a polynomial as again power of y in second term is not a whole number

v) $x^{10} + y^3 + t^{50}$

Sol :→ This is not a polynomial in one variable but a polynomial in 3 variable.

Q2 Write the coefficients of x^2 in each of the following:

i) $2 + x^2 + x$

Sol :→ Coefficient of x^2 is 1

ii) $2 - x^2 + x^3$

Sol : → Coefficient of $x^2 = -1$

iii) $\frac{\pi}{2}x^2 + x$

Sol : → Coefficient of $x^2 = \frac{\pi}{2}$

iv) $\sqrt{2}x - 1$

Sol :→ x^2 is not present hence no coefficient

Q3 Give one example each of a binomial of degree 35, and of a monomial of degree 100

Sol ⟹ $x^{35} + 5$ is a binomial of degree 35

$2y^{100}$ is a monomial of degree 100

Q4 Write the degree of each of the following polynomials:

i) $5x^3 + 4x^2 + 7x$

Sol : → Degree is 3 as highest power

ii) $4 - y^2$

Sol → Degree is 2

iii) $5t - \sqrt{7}$

Sol : → Degree is 1

iv) 3

Sol → Degree is 0

Q5 Classify the following as linear, quadratic and cubic polynomials:

i) x^2+x

Sol :→ quadratic polynomial

ii) $x-x^3$ → cubic polynomial

iii) $y+y^2+4$ → quadratic polynomial

iv) $1+x$ → linear polynomial

v) $3t$ → Linear polynomial

vi) r^2 → quadratic polynomial

vii) $7x^3$ → Cubic polynomial

Zero of a Polynomial

A zero or root of a polynomial function is a number that when plugged in for the variable, makes the function equal to zero.

Note :→ Every linear polynomial has one and only one zero

:→ A polynomial can have more than one zero e.g quadratic equations have 2 zero cubic have 3 zeroes

:→ A zero of polynomial need not to be zero.

:→ 0 may be a zero of polynomial.

EXERCISE 2.2

Q① find the value of polynomial $5x - 4x^2 + 3$ at

i) $x = 0$ ii) $x = -1$ iii) $x = 2$

Soli(i): ⟶ Let $P(x) = 5x - 4x^2 + 3$

Put $x = 0$

$\therefore P(0) = 5(0) - 4(0)^2 + 3$

$\Rightarrow P(0) = 0 - 0 + 3$

$\Rightarrow$ At $x = 0$ value of given $P(x) = 3$

ie $\boxed{P(0) = 3}$

ii) Now $P(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

$= -6$

$\therefore \boxed{P(-1) = -6}$

iii) $P(2) = 5(2) - 4(2)^2 + 3$

$= 10 - 16 + 3$

$\Rightarrow \boxed{P(2) = -3}$

Q2 Find $P(0)$, $P(1)$ and $P(2)$ for each of the following polynomials :⟶

i) $P(y) = y^2 - y + 1$

Sol $P(0) = (0)^2 - (0) + 1 = 1$

$P(1) = (1)^2 - (1) + 1 = 1$

$P(2) = (2)^2 - (2) + 1 = 3$

ii) $P(t) = 2 + t + 2t^2 - t^3$

Sol: $\longrightarrow$ $P(0) \longrightarrow 2 + 0 + 2(0)^2 - (0)^3$

$\rightarrow 2 + 0 + 0 - 0 = 2$

$P(1) = 2 + (1) + 2(1)^2 - (1)^3$

$= 2 + 1 + 2 - 1$

$P(1) = 4$

$P(2) = 2 + (2) + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8$

$P(2) = 4$

iii) $P(x) = x^3$

$P(0) = (0)^3 = 0$

$P(1) = (1)^3 = 1$

$P(2) = (2)^3 = 8$

iv) $P(x) = (x-1)(x+1)$

$P(0) = (0-1)(0+1)$

$= (-1)(+1)$

$P(0) = -1$ Ans//

$P(1) = (1-1)(1+1)$

$P(1) = 0$ Ans//

$P(2) = (2-1)(2+1)$

$P(2) = 3$ Ans//

Q3 Verify whether the following are zeroes of the polynomial indicated against them

i) $P(x) = 3x+1$, $x = -\frac{1}{3}$

Sol :⟶

$$P\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1$$

$$= -1 + 1$$

$$= 0 \quad \text{Ans//}$$

ii) $P(x) = 5x - \pi$, $x = \frac{4}{5}$

Sol:⟶ $P\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi$

$$= 4 - \pi \quad \text{Ans//}$$

iii) $P(x) = x^2 - 1$, $x = 1, -1$

$$P(1) = (1)^2 - 1$$

$$= 1 - 1 = 0 \quad \text{Ans//}$$

$$P(-1) = (-1)^2 - (1)$$

$$= 1 - 1 = 0 \quad \text{Ans//}$$

iv) $P(x) = (x+1)(x-2)$, $x = -1, 2$

$$P(-1) = (-1+1)(2-2)$$

$$= 0 \quad \text{Ans//}$$

$$P(2) = (2+1)(2-2)$$

$$= 3(0)$$

$$= 0 \quad \text{Ans//}$$

v) $P(x) = x^2$, $x = 0$

$$P(0) = (0)^2$$

$$= 0 \quad \text{Ans//}$$

vi) $P(x) = lx+m,\ x = -\frac{m}{l}$

Sol :→ $P\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = -m + m = 0$

vii) $P(x) = 3x^2 - 1,\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

$P\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$

$= 3 - 1 = 2$

$P\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$

$3(4) - 1 = 12 - 1 = 11$ Ans//

viii) $P(x) = 2x+1,\ x = \frac{1}{2}$

$P\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$ Ans//

Q4 Find the zero of the polynomial in each of the following cases.

i) $P(x) = x+5$

Sol Put $P(x) = 0 \Rightarrow x+5 = 0 \Rightarrow x = -5$

$\therefore$ -5 is zero of given polynomial

ii) $P(x) = x-5$

Sol:→ Put $P(x) = 0 \Rightarrow x-5 = 0 \Rightarrow x = 5$

$\therefore$ $+5$ is zero of given $P(x)$

iii) $P(x) = 2x+5$

Put $P(x) = 0 \Rightarrow 2x+5 = 0 \Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$

$\therefore$ $-\frac{5}{2}$ is zero of given $P(x)$

by $x-a$, then quotient is $q(x)$ and the remainder is $r(x)$ ie

$$P(x) = (x-a)\,q(x) + r(x)$$

as Dividend = (Divisor × Quotient) + Remainder

Since the degree of $x-a$ is 1 and the degree of $r(x)$ is less than the degree of $x-a$, the degree of $r(x)$ $=0$, This means that $r(x)$ is constant say r.

So for every value of x, $r(x) = r$

Therefore $$P(x) = (x-a)\,q(x) + r$$

In particular, if $x = a$, this equation gives

$$P(a) = (a-a)\,q(a) + r$$

$$= r$$

which proves theorem.

e.g :⟶ Find the remainder when $x^4+x^3-2x^2+x+1$ is divided by $x-1$

Solution :⟶ Here $P(x) = x^4+x^3-2x^2+x+1$ and zero of $x-1$ is 1

So $P(1) = (1)^4 + (1)^3 - 2(1)^2 + 1 + 1$

$= 2$

So by remainder Theorem 2 is the remainder when the given $P(x)$ is divided by $(x-1)$

EXERCISE 2·3

Q1 Find the remainder when x^3+3x^2+3x+1 is divided by

i) $x+1$

Sol :⟶ Here $P(x) = x^3+3x^2+3x+1$ and zero of $x+1 = -1$

$\therefore P(-1) = (-1)^3+3(-1)^2+3(-1)+1$

$= -1+3-3+1$

$= 0$

$\therefore$ Remainder is 0

By division method :⟶

$$x+1 \;\overline{)\; x^3+3x^2+3x+1 \;(\; x^2+2x+1}$$

$$\underline{-(x^3+x^2)}$$

$$2x^2+3x+1$$

$$\underline{-(2x^2+2x)}$$

$$x+1$$

$$\underline{-(x+1)}$$

$$0$$

Hence remainder = 0

ii) $x-\frac{1}{2}$

Sol Here $P(x) = x^3+3x^2+3x+1$ and zero of $x-\frac{1}{2}$ is $\frac{1}{2}$

$$P\left(\tfrac{1}{2}\right) = \left(\tfrac{1}{2}\right)^3+3\left(\tfrac{1}{2}\right)^2+3\left(\tfrac{1}{2}\right)+1$$

$$= \frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1 = \frac{1+6+12+8}{8} = \frac{27}{8}$$ Ans//